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HEAT EXCHANGER REGULATION BASED ON METHOD OF INVARIANT ELLIPSOIDS

Vojislav Filipovic[1], Dragutin Lj. Debeljkovic[2], Milan Matijevic[1] &Vladimir Đorđevic[1]

[1]University of Kragujevac, Serbia

[2]Faculty of civil engineering, Megatrend University of Belgrade, Serbia

Key words

Heat exchangers
uniform bounded disturbance
invarinat ellipsoids
optimal regulator
convex optimization
solvers
YALMIP
SeDuMi

Abstract

The paper proposes a new method for heat exchanger regulation. At the first, a lumped parameter mathematical model for heat exchanger is proposed. The model is linear and includes a disturbance which can be categorized as an almost arbitary disturbance. Overall a priori knowledge about disturbance is contained in the fact that the disturbance is uniformly bounded. Regulator design is based on (in discrete case) or (in continuous case) optimisation. These methodologies belong to the class of serious problems in control engineering. In this paper is used the latest results based on method of invarinat ellipsoids which relax the above problems. The problem of regulator design comes down to minimizing the size of the invariant ellipsoids of the dynamic system. Numerical solution isbased on application of convex analysis (semidefinite programming and linear matrix inequalities). To solve these problems, there are powerful solvers (YALMIP and SeDuMi) so practical regulator implementation is significantly simplified. Intensive simulations give verification of regulator efficiency and its superiority over the well-known LQ regulator. Energy efficiency and quality of HVAC system is in strong correlation with regulator performances.

1. INTRODUCTION

In recent years, Heating, Ventilation and Air Conditioning (HVAC) systems integrated into building automation systems have become very popular.

Their popularity results from their ability to quickly set and retain demanded temperature by using various sensors in combination with a sophisticated feedback-control system. In addition, efficient control strategies play an essential role in developing improved energy control systems for buildings. The most important criteria for designing HVAC plants are energy efficiency and indoor climate conditions. Heat exchangers are standard components within HVAC systems [1-3]. To improve energy efficiency, there is a growing interest in developing techniques that compute control signals that minimize energy consumption. Such control techniques require a model of heat exchangers that relates the control signals to the space temperature. The mathematical model based on the fundamental principles is in the form of partial differential equations [4]. Regulator design, in this case, is a non-trivial task [5]. The alternative is obtaining and exploitation of models with lumped parameters [6].

In this paper we use the last option for mathematical model of heat exchangers. Under real conditions there are temperature variations as well as heating fluid flow variations. Such phenomena will be modeled as uniformly bounded disturbances. This is a deviation from the dominant assumption that the disturbances are stochastic signals. From point of view of practical application it is very important that required a priori information about disturbances is minimal.

Regulator design based on models of uniformly bounded disturbances belongs to a set of the very difficult problems in control engineering. In the case of discrete models, the problem is solved in [7-8] (l^1 optimization). In continuous domain, the problem is solved in [9-10] (L^1 optimization). By aforementioned methodology, results are regulators with very high order.

In continuous domain, which is discussed in this paper, infinitely dimensional regulator is result of the L^1 norm minimization. These facts prevent the application of l^1 and L^1 the optimal regulators.

The latest studies of regulator design in the presence of bounded disturbances access from positions of invariant sets [11].

Invariant ellipsoid as a special form of the invariant set is considered, which significantly facilitates troubleshooting [12-13], which consists in minimizing the size of the invariant ellipsoid of dynamic system.

The original L^1 optimization comes down to the use of LMI (Linear Matrix Equation) [14], and semidefinite programming [15]. For these methodologies, there are very efficient solvers (YALMIP, SeDuMi, CVX) and they will be used in the simulations.

A new model of the heat exchanger is proposed by this paper.

The model is linear and includes bounded disturbance. The linear part of the model is second order in derivatives.

Heat exchanger description is provided in the state space form. Applying the method of invariant ellipsoids the regulator is designed.

The regulator has a simple structure and is obtained using modern solvers based on the convex analysis [16]. At this manner designed regulator is a step towards energy savings.

The last part of the paper presents simulation results concerning of the heat exchanger regulation.

2. THE CONCEPT OF INVARIANT ELLIPSOIDS FOR DYNAMICAL SYSTEMS

Suppose that the LTI (Linear Time Invariant) dynamic system is described by the continuous time stationary linear system.

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + D\mathbf{w}(t)$$

(1)

$$\mathbf{y}(t) = C\,\mathbf{x}(t)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the vector of the system phase state, $\mathbf{y}(t) \in \mathbb{R}^l$ is the vector of the system output, $\mathbf{w}(t) \in \mathbb{R}^m$ are the exogenous disturbances bounded at each time instant,

$$\left\|\mathbf{w}(t)\right\| \le 1, \ \ \forall t \ge 0 \qquad (2)$$

where $\|\cdot\|$ is the Euclidean norm of the vector.

Therefore, we consider the L^∞ bounded exogenous disturbances $w(t)$.

$$\left\|\mathbf{w}(t)\right\|_\infty = \sup_{t\ge0}\sqrt{\mathbf{w}^T(t)\mathbf{w}(t)} \ \le 1 \qquad (3)$$

We assume that system (1) is stable, that is, A is Hurwitzian matrix with negative real parts, the pair (A, B) is controllable, and C is the maximum-rank matrix.

We determine the family of invariant ellipsoids of this system.

The ellipsoid $\mathcal{E}_x$ with the center at the origin

$$\mathcal{E}_x = \left\{\mathbf{x}(t) \in \mathbb{R}^n : \ \mathbf{x}^T(t)P^{-1}\mathbf{x}(t) \le 1\right\}, \ P > 0 \qquad (4)$$

is invariant to the variable $\mathbf{x}(t)$ (in state) for dynamic system (1)-(2), if it follows from the condition $\mathbf{x}(0) \in \mathcal{E}_x$ that $\mathbf{x}(t) \in \mathcal{E}_x$ for all time instants $\forall t \ge 0$.

P is called the matrix of the ellipsoid $\mathcal{E}_x$.

The ellipsoid invariant to the variable $\mathbf{y}(t)$, that is, system output, is determined from (1) and (4) by

$$\mathcal{E}_y = \left\{\mathbf{y}(t) \in \mathbb{R}^m : \ \mathbf{y}^T(t)\left(CPC^T\right)^{-1}\mathbf{y}(t) \le 1\right\} \qquad (5)$$

The invariant ellipsoids may be regarded as the characteristic of the impact of the exogenous disturbances on the trajectories of the dynamic system.

Since the invariance of the ellipsoid $\mathcal{E}_y$ and its size are the measure of disturbances $\mathbf{w}(t)$ effects on the system outputs $\mathbf{y}(t)$, a natural criterion for regulator synthesis is to minimize the ellipsoid $\mathcal{E}_y$.

In this paper it will be used the criterion

$$f(P) = tr\left[CPC^T\right] \qquad (6)$$

which is the sum of squares of axis of the invariant ellipsoid.

3. HEAT EXCHANGER MODEL IN THE STATE SPACE

The tubular counter-current heat exchanger is presented on Fig. 1.

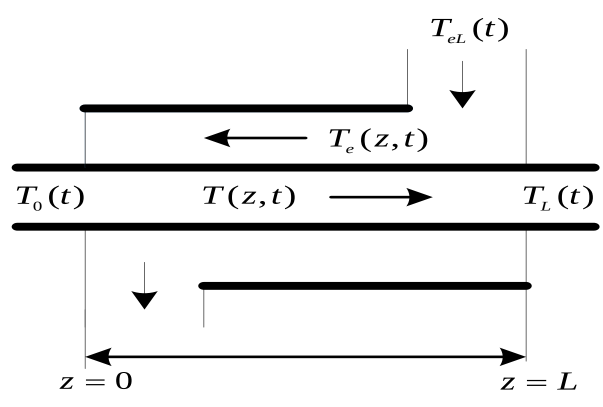

Figure 1: Counter-current heat exchanger

The short description of the heat exchanger follows [17].

A liquid fluid of constant density ρ and heat capacity c_p flows through the internal tube of length L with **constant**velocity w. Input fluid has temperature T_0.

This fluid exchanges heat with the **second liquid** of constant density ρ_e and the heat capacity c_{pe} which flows counter-currently in the jacket with a time varying velocity $w_e(t)$.

The input temperature of this fluid is T_e and output inter fluid temperature is T_L.

Both temperatures depend on time and special position along the tube.

The dynamic of heat exchanger can be described with two partial differential equations.

$$\frac{\partial T(z,t)}{\partial t}+w\frac{\partial T(z,t)}{\partial z}=\wp\left(T_e(z,t)-T(z,t)\right) \qquad (7)$$

$$\frac{\partial T_e(z,t)}{\partial t}-w_e(t)\frac{\partial T_e(z,t)}{\partial z}=\wp_e\left(T(z,t)-T_e(z,t)\right) \qquad (8)$$

where the coefficients $\wp$ and $\wp_e$ are known and defined by geometry of heat exchangers and its fluid characteristics.

Regulator design and implementation, based on the model (7) - (8), is a very complex problem.

Because of that, in this paper, the heat exchanger will be considered as a process with lumped parameters.

In accordance with [18] fluid that is heated can be regarded as a disturbance.

The change of its flow occurs as a disturbance.

Process diagram of the heat exchanger is shown in the following figure

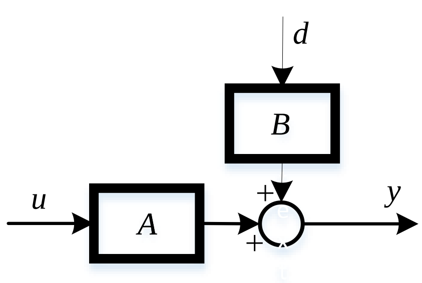

Figure 2: Process diagram of the heat exchanger.
A – fluid for heating of process fluid.
B – process fluid.

At Fig.2, the input signal $u(t)$ represents the flow of heating fluid, disturbance $d(t)=w(t)$ is a change of flow rate of process fluid (i.e. heated fluid), and $y(t)$ is the temperature of the heated process fluid.

Experimental studies in [6] have showed the following

a) in the case of constant flow of process fluid (heated fluid), input $u(t)$ - flow of heating fluid – output $y(t)$ - the temperature of the heated process fluid) model is linear

b) in the case of constant flow of heating fluid $u(t)$, under changes of flow rate of process fluid (i.e. heated process fluid, disturbance input $d(t)=w(t)$, output $y(t)$ - the temperature of the heated process fluid) model is nonlinear

Based on the experiment [6], it is concluded that the model on Fig.2. consists of linear block A and nonlinear block B (correct nonlinear model is Hammerstein model).

Nonlinear Hammerstein model is BIBO (bounded input bounded output) stable [19]. Since disturbances (*d*) are uniformly bounded then the output of Hammerstein model (*w*) is bounded.

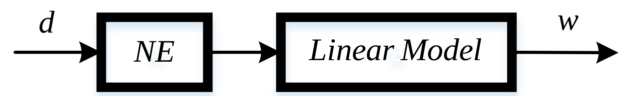

Figure 3: Hammerstein model of disturbance effect on system output

Based on Fig.2 and Fig.3, the final model of the heat exchanger is obtained and shown on Fig.4.

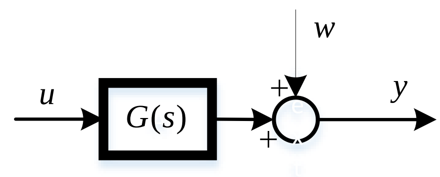

Figure 4: Heat exchanger model

In [6] transfer function $G(s)$ (see Fig.4) is identified as

$$G(s)==\frac{-65.22s-11.23}{s^2+2.664s+0.604}=-\frac{65.22s+11.23}{s^2+2.664s+0.604} \qquad (9)$$

Problem of regulator synthesis based on plant model presented in Fig.4 with uniformly bounded disturbance we will solve using methodology of invariant ellipsoids [12].

In order to apply methodology of invariant ellipsoids it is necessary to transfer the model (9) into the state space model.

$$\dot{\mathbf{x}}(t)=A\mathbf{x}(t)+B_1\mathbf{u}(t) \qquad (10)$$

$$\mathbf{y}(t)=C\mathbf{x}(t)+B_2\mathbf{u}(t) \qquad (11)$$

where:

$$A=\begin{pmatrix}-2.664 & -0.604\\ 1.000 & 0.000\end{pmatrix},\quad B_1=\mathbf{b}_1=\begin{bmatrix}1\\0\end{bmatrix},\quad C=\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix},\quad B_2=\mathbf{b}_2=\begin{bmatrix}1\\0\end{bmatrix}$$

According to Fig.4 complete model of heat exchanger has the form

$$\begin{aligned}\dot{\mathbf{x}}(t)&=A\mathbf{x}(t)+\mathbf{b}_1u(t)+\mathbf{d}w(t)\\ \mathbf{y}(t)&=C\mathbf{x}(t)+\mathbf{b}_2u(t)\end{aligned} \qquad (12)$$

$$\mathbf{d}=\begin{bmatrix}0\\1\end{bmatrix} \qquad (13)$$

It should be noted, a well-known fact, that there is freedom (not uniqueness) in the selection of the second relation in (12).

The key feature of the model (12) - (13) that there is only information about disturbances that ones are bounded.

$$\left\|\mathbf{w}(t)\right\| \le 1, \ \forall t \ge 0 \quad (14)$$

Such a description of the disturbances is extremely general which is very important for practice. It belongs to the class of almost arbitrary disturbances [20].

The condition (14) defines methodology of regulator design, which is a very difficult problem for the given case of application. Methodology is exposed in the next chapter.

4. REGULATOR DESIGN BASED ON THE METHODOLOGY OF INVARIANT ELLIPSOIDS

In this section, the regulator design approach will be considered under conditions of persistent disturbance based on the methodology of invariant ellipsoids and linear matrix inequalities. State feedback regulator will be determined. The basic requirement is that the designed regulator performs stabilization, and that, in turn, minimizes the corresponding invariant ellipsoid.

A linear continuous system is described in the following way

$$\dot{x}(t) = Ax(t) + B_1u(t) + Dw(t), \ \ x(0) = x_0 \quad (15)$$

$$y(t) = Cx(t) + B_2u(t) \quad (16)$$

where are $A \in \mathbb{R}^{n\times n}$, $B_1 \in \mathbb{R}^{n\times 1}$, $D \in \mathbb{R}^{n\times 1}$, $B_2 \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{n\times n}$ corresponding matrices and vectors, $x(t) \in \mathbb{R}^{n}$ is the system phase state, $y(t) \in \mathbb{R}^{n}$ is the system output, $u(t) \in \mathbb{R}^{1}$ is the control, and $w(t) \in \mathbb{R}^{1}$ is the exogenous (uniform limited) disturbance satisfying the constraint

$$\left\|w(t)\right\| \le 1, \ \forall t \ge 0 \quad (17)$$

where $\|\cdot\|$ Euclidean norm of the vector. It is also required that the

a) (A, B_1) is controllable pair

b) $B_2^T C = 0$

It is not necessary that the matrix A is Hurwitzian which means that the regulator can be applied to unstable systems.

The fundamental elements of the theory are exposed in [12]. The regulator has the form

$$u(t) = Kx(t) \quad (18)$$

The system with closed feedback loop (15) - (18) has the following form

$$\dot{x}(t) = (A + B_1K)x(t) + Dw(t) \quad (19)$$

$$y(t) = (C + B_2K)x(t) \quad (20)$$

The problem of designing a static regulator by state (18) which rejects optimally (in the sense of the trace that is output-invariant to the ellipsoid) the exogenous disturbances is equivalent to that of minimization of

$$tr\left[CPC^T + B_2ZB_2^T\right] \to \min \quad (21)$$

under constraints

$$AP + PA^T + \alpha P + B_1Y + Y^TB_1^T + \frac{1}{\alpha}DD^T \le 0, \ \ \alpha > 0 \quad (22)$$

$$\begin{bmatrix} Z & Y \\ Y^T & P \end{bmatrix} \ge 0 \quad (23)$$

$$P > 0 \quad (24)$$

The regulator (18) is obtained as

$$\hat{K} = \hat{Y}\hat{P}^{-1} \quad (25)$$

where $\hat{P}$, $\hat{Y}$ and $\hat{Z}$ the solution of the problem (21) - (24)

Minimum invariant ellipsoid is

$$C\hat{P}^TC^T + B_2\hat{Z}B_2^T \quad (26)$$

The graphical representation of control system is given on the next figure

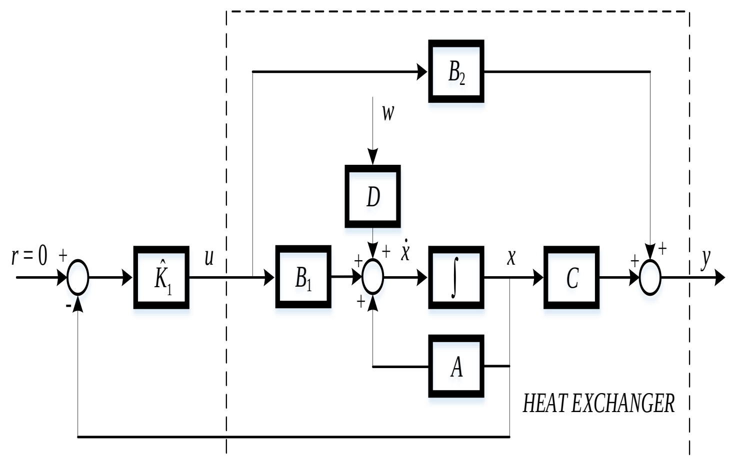

Figure 5: Control system for heat exchanger ($\hat{K}_1 = -\hat{K}$)

Remark 1. The condition $B_2^T C = 0$ can be avoided [21]. In this case, the criterion (21) se is replaced by the following

$$tr\left[CPC^T + CY^TB_2^T + B_2YC^T + B_2ZB_2^T\right] \to \min \quad (27)$$

In other details, the new procedure is identical to the procedure (21) - (24).

Remark 2. The presence of $B_2u(t)$ in relation (16) provides, in minimizing the output signal, avoiding large control signals. It follows that the matrix B_2 has the same role as the matrix S in LQ regulator design

$$J = \int_0^{\infty} \left(x^T(t) R x(t) + u^T(t) S u(t) \right) dt \qquad (28)$$

The alternative is to explicitly introduce limitations to the control signal.

5. SIMULATION RESULTS AND DISCUSSION

Let consider the model (12)-(13) where A, B_1, C and B_2 the same as in the relations (10)-(11). It is easy to verify that (A, B_1) is controllable pair and that $B_2^T C = 0$. Using the optimal regulator $\hat{K}$; which is obtained by solving of relations (21)-(24), the invariant ellipsoid of output is minimized. In order to solve problem (21)-(24), it is used software packages SeDuMi and YALMIP based on MATALB software package. Numerical problem is solved by use of semidefinite programming (21) under constraints (22)-(24).

In the simulations is very important, prior to calculating the vector regulator gain, to define parameter α ($\alpha > 0$). For this purpose is defined segment $\alpha \in [\varepsilon, k_\alpha]$, for $\exists \varepsilon > 0$ and $k_\alpha > 0$. Simulations are conducted for $k_\alpha \le 5$. The segment $[\varepsilon, k_\alpha]$ is subdivided into the collection of subsegments, and by YALMIP is searched value α for that value is obtained minimum value criteria

$$tr(C\hat{P}^T C^T + B_2 \hat{Z} B_2^T)$$

If the lowest value α is on the border segment $[\varepsilon, k_\alpha]$, then the segment should be extended until within the segment finds α for that the above criteria is minimal. In the first step is carried out a rough division of the segment $[\varepsilon, k_\alpha]$. When it finds the best value α, then defines a small segment containing α, and the search procedure for the best α repeats with a finer division of this segment. At this manner, relatively quickly, we can receive value α.

Below we will illustrate the behavior of the closed-loop system under various disturbances.

A) The disturbance is given by $w(t) = \sin(t/2)$. In this case the solution of the problem (21) - (24) is

$$\alpha = 1.1569, \ \hat{K} = [-0.9035 \quad -2.2924]$$

On Fig.6 and Fig.7 it is shown the minimum invariant output ellipse of the system (Fig.6), and both control and disturbance signal (Fig.7)

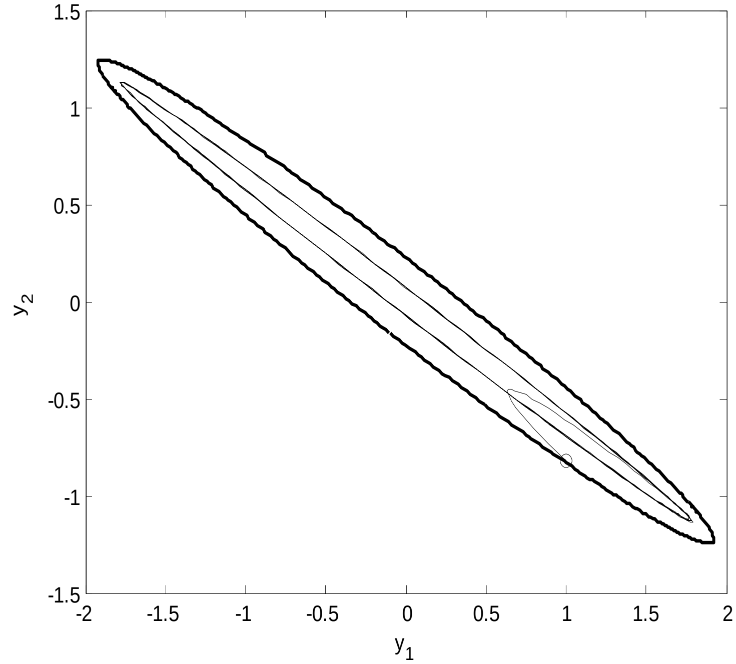

Figure 6: Minimum invariant output ellipse for $B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $w(t) = \sin\frac{t}{2}$.

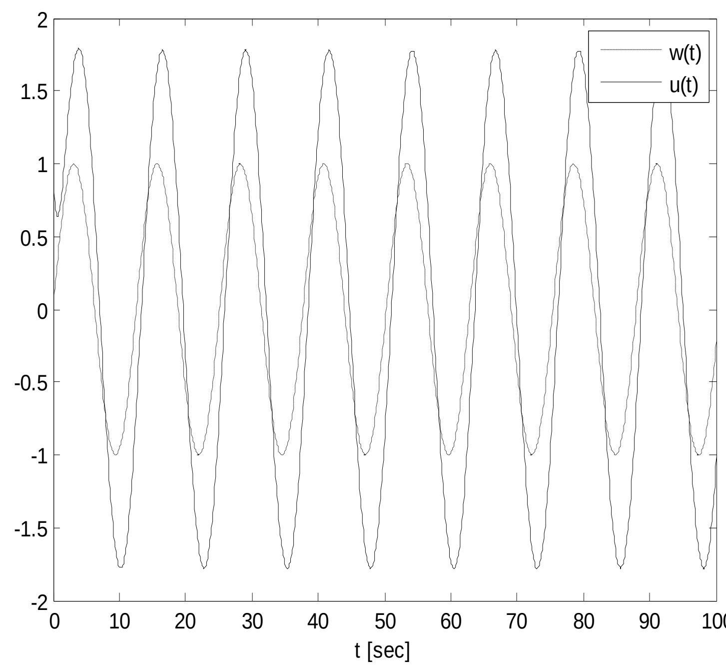

Figure 7: Control $u(t)$ and disturbance $w(t)$

In accordance to remark 2, using the vector B_2, it can be influenced on the control signal magnitude. If the vector B_2 is adopted as $B_2^T = [2.5 \quad 0]$, for solution of the problem (21) - (24) is obtained

$$\alpha = 0.3716, \ \hat{K} = [-0.1197 \quad -0.3037]$$

and behavior of the system is shown on Fig.8 and Fig.9.

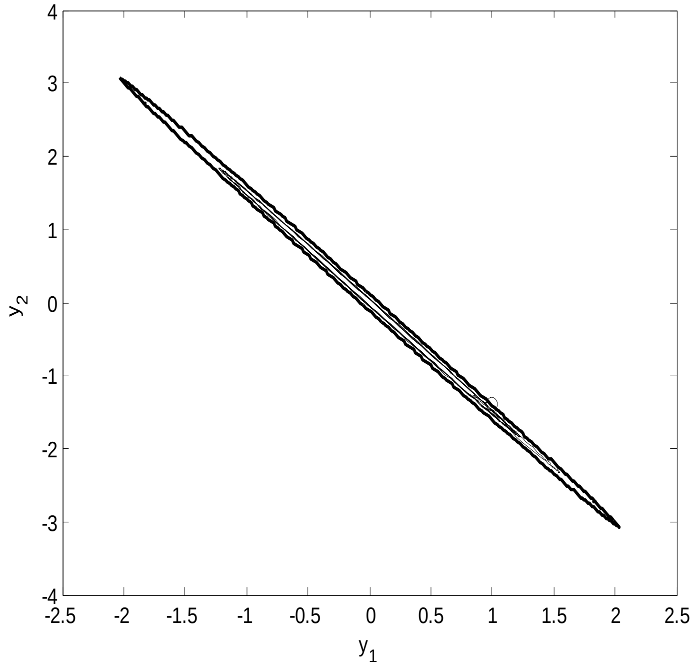

Figure 8: Minimum invariant output ellipse for $B_2^T = \begin{bmatrix} 2.5 & 0 \end{bmatrix}$.

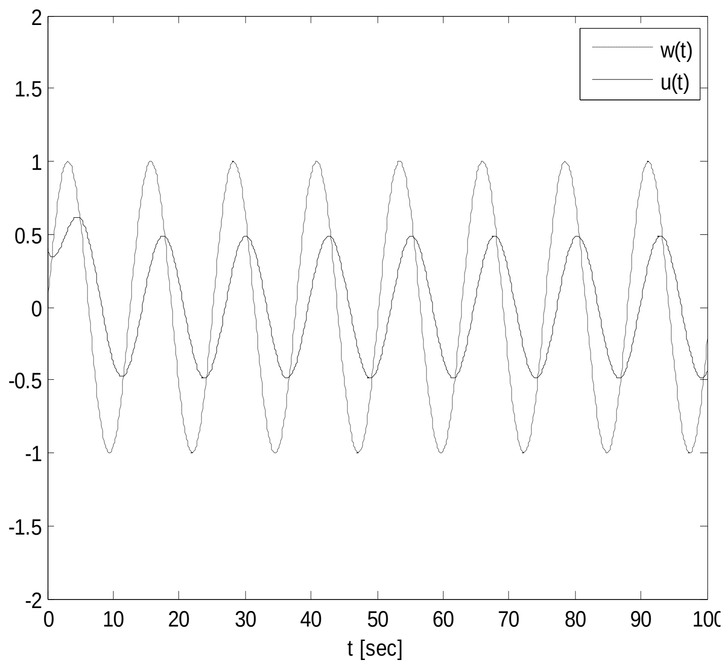

Figure 9: Control $u(t)$ and disturbance $w(t)$

With Fig.8 can be seen how is increased the coordinate y_2, and with Fig.9 how is reduced amplitude of the control signal $u(t)$. If the vector B_2 is adopted as $B_2^T = \begin{bmatrix} 0.1 & 0 \end{bmatrix}$, for solution of the problem (21) - (24) is obtained

$$\alpha = 4.3284,\ \hat{K} = \begin{bmatrix} -6.5375 & -24.6378 \end{bmatrix}$$

The behavior of the system is shown in Fig.10 and Fig.11

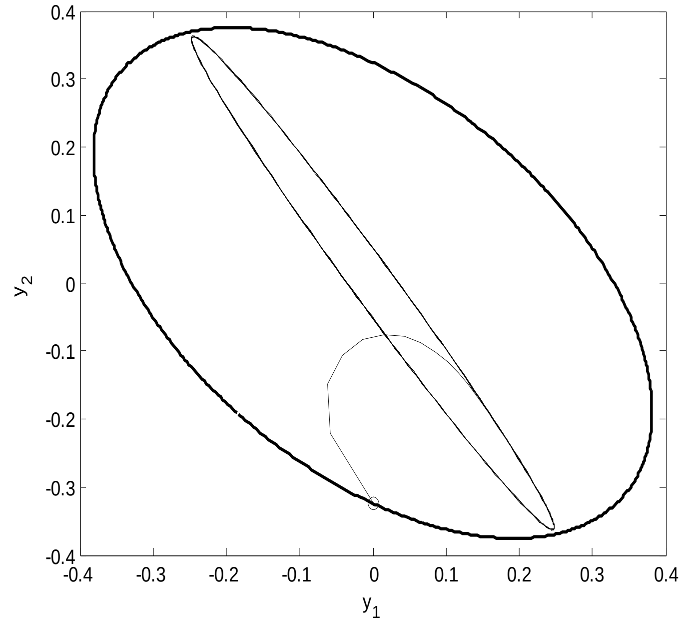

Figure 10: Minimum invariant output ellipse for $B_2^T = \begin{bmatrix} 0.1 & 0 \end{bmatrix}$.

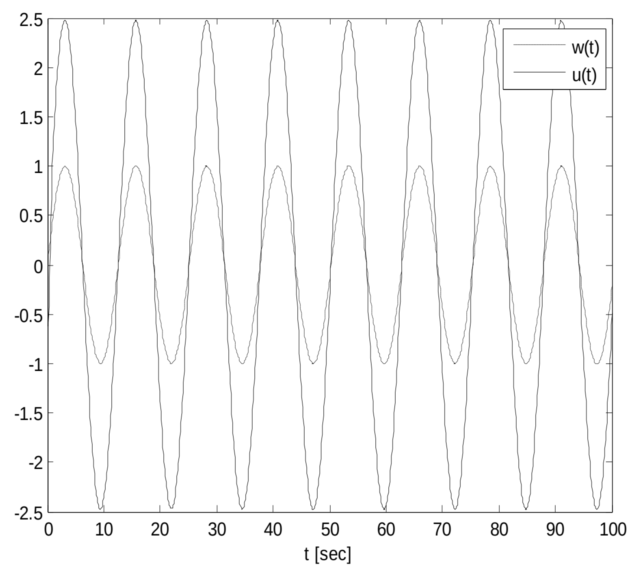

Figure 11: Control $u(t)$ and disturbance $w(t)$ for $B_2^T = \begin{bmatrix} 0.1 & 0 \end{bmatrix}$

With Fig.8 it can be seen how is decreased size of the invariant output ellipse, which is convenient from a position of control. But, on the other hand, with Fig.11. it can be seen that is significantly increased the amplitude of control signal, which is disadvantageous.

B) The disturbance is given by $w(t) = \mathrm{sgn}(\sin(t/2))$. In this case, the solution of the problem (21) - (24) is

$$\alpha = 1.1569,\ \hat{K} = \begin{bmatrix} -0.9035 & -2.2924 \end{bmatrix}$$

The behavior of the system is shown in Fig.12 and Fig.13.

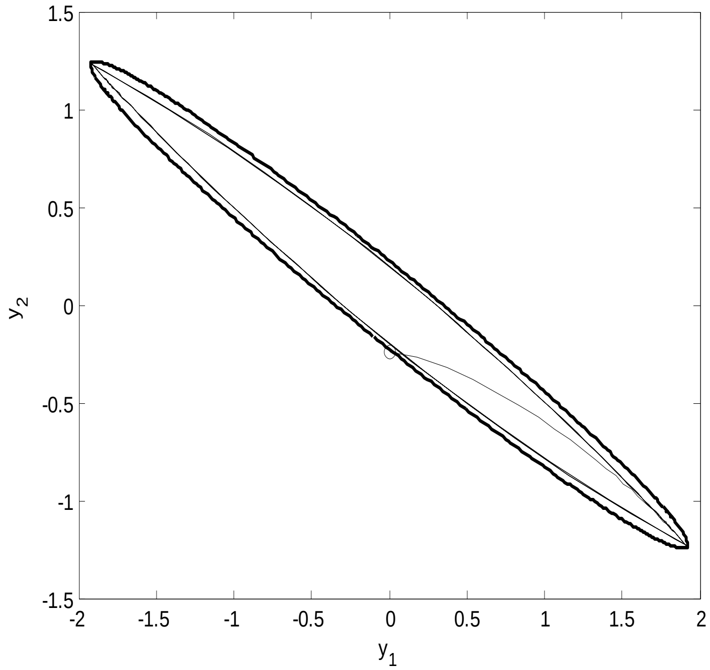

Figure 12: Minimum invariant output ellipse

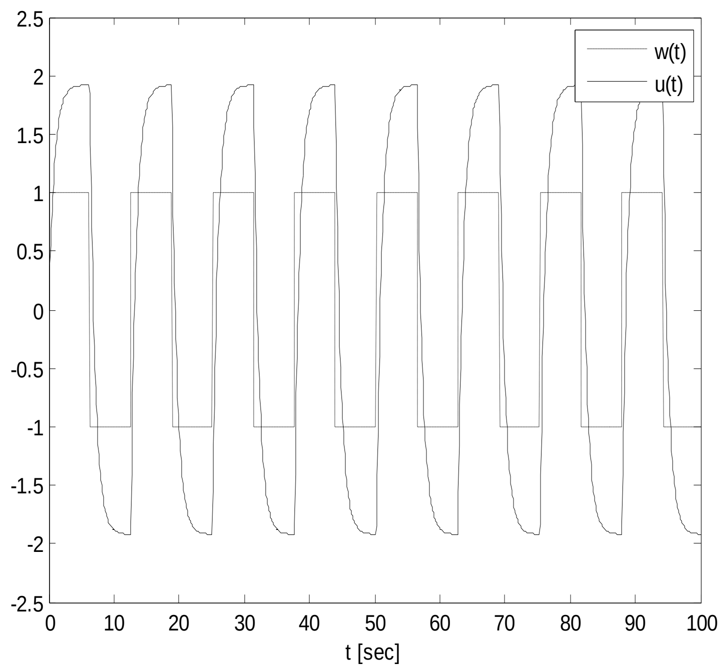

Figure 13: Control $u(t)$ and disturbance $w(t)$

The simulations also show that by changing the vector B_2 it is obtained similar results as in the case under A

C) Disturbance $w(t)$ is step function, and in that case

$$\alpha = 1.1569, \ \hat{K} = \begin{bmatrix} -0.9035 & -2.2924 \end{bmatrix}$$

Minimum invariant output ellipse is shown at Fig.14, and disturbance $w(t)$ and control signal $u(t)$ are shown at Fig.15.

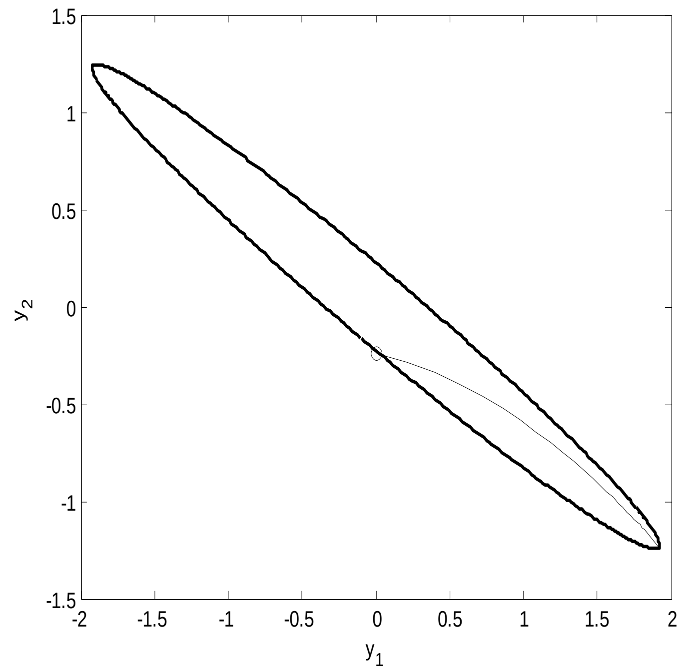

Figure 14: Minimum invariant output ellipse

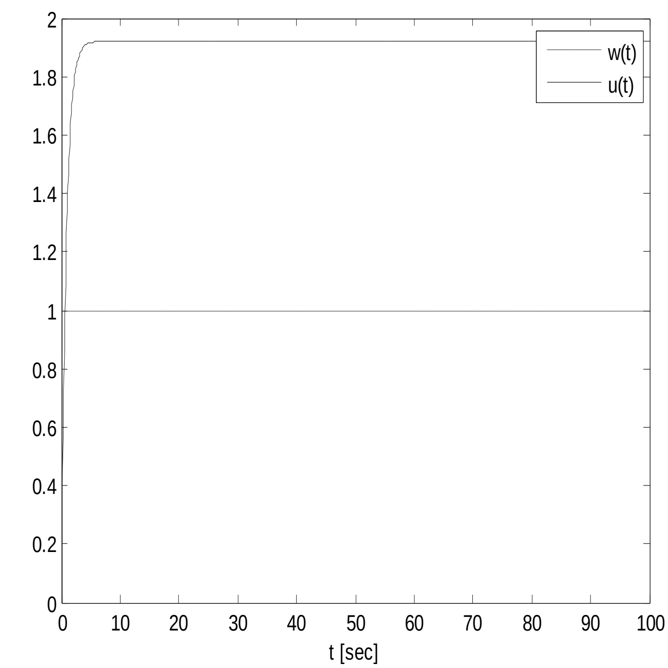

Figure 15: Control $u(t)$ and disturbance $w(t)$

The behavior of the state vector is shown in Fig.16.

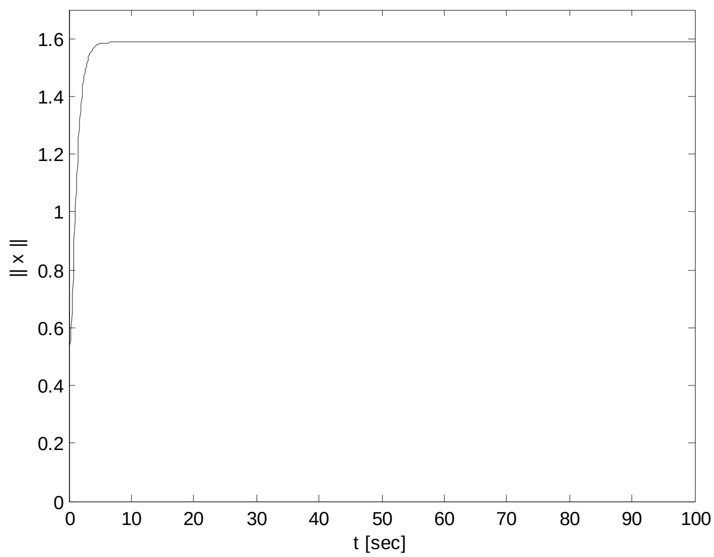

Figure 16: Norm of the system state vector

D) Disturbance $w(t)=0$, and in tator based on methodology of invariant ellipsoids (for system (15)-(16)) is

$$\alpha = 1.1569,\ \hat{K} = \begin{bmatrix} -0.9035 & -2.2924 \end{bmatrix}$$

For the case $w(t)=0$ and system (15)-(16), optimal regulator is LQ regulator, which can be obtained by minimisation of functional (28). For adopted matices in (28)

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ i } S = 1.6$$

LQ regulator is defined by

$$K_{LQ} = \begin{bmatrix} -0.251 & -0.3909 \end{bmatrix}$$

Comparison of the above regulator is given in the following figure 17.

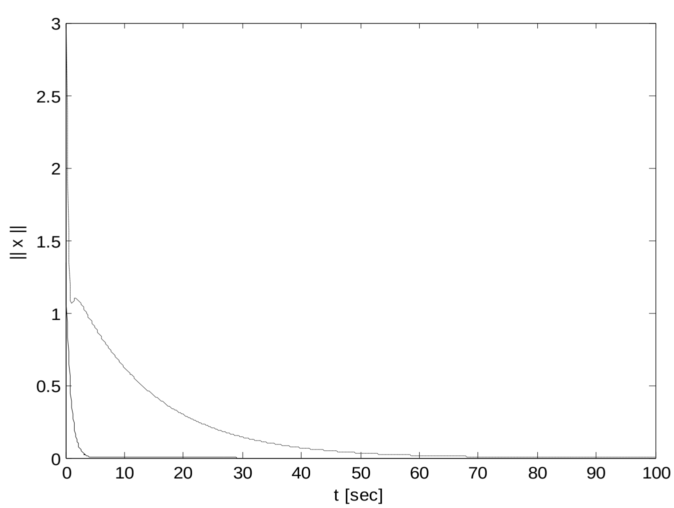

Figure 17: Norm of the system state vector

Legend:

——— *Regulator based on methodology of invariant ellipsoids*

- - - - - *LQ regulator*

It can be seen that the dynamic behavior of the system with regulator based on the ideology of invariant ellipsoid is significantly superior in comparision to behavior of the system with LQ regulator. This can be explained by the higher values of the elements of the vector $\hat{K}$ in comparison to the LQ gain vector K_{LQ}.

6. CONCLUSIONS

The paper discusses the problem of regulator design for linear systems under effects of uniformlybounded disturbances. This assumption about the external disturbances is very general and embraces the minimum a priori information about ones. It is very realistic and very important for practical applications. The problem of regulator synthesis comes down on the minimization of the specified criteria (semidefinite programming) under constraints in the form of linear matrix inequalities. Intensive simulations for different types of disturbances verify good regulator properties. In the case of disturbance absence, regulator based on the methodology of invariant ellipsoid has superior characteristics compared to LQ regulator. Potentials for further work on the problem are: i) extension of problems on discrete case, ii) the design of the regulator with L^{∞} limit on the control signal.

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